KAIST-TH 12/96
CBNU-TH 960707
SNUTP 96-074
hep-ph/9608222

# Cosmological Implications of Radiatively Generated Axion Scale

Kiwoon Choi

*Department of Physics, Korea Advanced Institute of Science and Technology*
*Taejon 305-701, Korea*

Eung Jin Chun

*Department of Physics, Chungbuk National University*
*Cheongju, Chungbuk 360-763, Korea*

Jihn E. Kim

*Department of Physics and Center for Theoretical Physics*
*Seoul National University, Seoul 151-742, Korea*

**Abstract**

We study cosmological implications of supersymmetric axion models in which the axion scale is generated radiatively. Such models lead to the so-called thermal inflation and subsequent reheating should be constrained not to yield a too large axion energy density at the time of nucleosynthesis. We examine how plausible it is that this nucleosynthesis constraint is satisfied for both hadronic and Dine-Fischler-Srednicki-Zhitnitskii type axion models. Baryogenesis and the possibility for raising up the cosmological upper bound on the axion scale in thermal inflation scenario are also discussed.

One of the attractive solutions to the strong CP problem is to introduce an anomalous Peccei-Quinn (PQ) symmetry $U(1)_{PQ}$ [1]. This solution predicts a pseudo-Goldstone boson, the (invisible) axion [2, 3], whose decay constant $F_a$ is tightly constrained by astrophysical and cosmological arguments. The allowed band of the axion scale $F_a$ lies between $10^{10}$ GeV and $10^{12}$ GeV [4] which is far away from the already known two mass scales, the electroweak scale and the Planck scale $M_P = 1/\sqrt{8\pi G_N}$. It is certainly desirable that this intermediate scale appears as a dynamical consequence when the known mass scales are set up in the theory. This indeed happens [5] in some class of spontaneously broken supergravity models which are commonly considered as the underlying structure of the supersymmetric standard model. Such models typically contain two basic mass scales, $M_P$ and the scale of local supersymmetry breaking $M_S$ in the hidden sector leading to $m_{3/2} = M_S^2/M_P = 10^2 \sim 10^3$ GeV. Supergravity interactions then generate soft supersymmetry breaking terms in the supersymmetric standard model sector which are of order $m_{3/2}$. In this scenario, radiative corrections to the Higgs doublet mass-squared associated with the large top quark Yukawa coupling can naturally lead to the electroweak symmetry breaking at the scale $M_W \simeq m_{3/2}$. When the PQ fields which are responsible for the spontaneous violation of $U(1)_{PQ}$ correspond to flat directions of the model, the intermediate axion scale $F_a$ can also be radiatively generated in terms of $M_P$ and $m_{3/2}$. In such a scheme, as was recently emphasized, the early universe experiences the so-called thermal inflation and subsequently a period dominated by coherently oscillating flaton fields [6]. The aim of this paper is to examine cosmological implications of PQ flatons in supergravity models with a radiative mechanism generating the axion scale.

One possible cosmological consequence of PQ flatons is the impact on the big-bang nucleosynthesis through their decay into axions. In the scheme under consideration, PQ flatons have generally order-one coupling to the Goldstone boson (the axion) in the unit of $1/F_a$ [7]. As we will argue later, axions produced by decaying flatons are hardly thermalized. In this paper, we first consider the energy density of these unthermalized axions at the time of nucleosynthesis together with its implications for both hadronic axion models [2] and Dine-Fischler-Srednicki-Zhitnitskii (DFSZ) type models [3]. Even when one takes a rather conservative limit on the axion energy density, this consideration provides a meaningful restriction for generic hadronic axion models and also for DFSZ type models with a rather large flaton mass.

As another cosmological implications of PQ flatons, we consider the possibility of rais-

ing up the cosmological upper bound on the axion scale $F_a$ through the late time entropy production [8] by oscillating PQ flatons. We argue that $F_a$ can be pushed up to about $10^{15}$ GeV without any cosmological difficulty in thermal inflation scenario. Finally, we point out that the Dimopoulos-Hall (DH) mechanism [9] for a late time baryogenesis can be naturally implemented in thermal inflation scenario. In the conclusion, we note that the case of $n = 2$ or 3 [see Eq. (3) below] provides a very concordant cosmological scenario.

We begin by describing how the intermediate axion scale can be radiatively generated in supergravity models in which the PQ fields correspond to flat directions. Let us consider a variant of the model of Ref. [5] with superpotential

$$W = k\frac{\phi_1^{n+2}\phi_2}{M_P^n} + h\frac{\phi_1^{n+1}H_1H_2}{M_P^n} + h_N NN\phi_1 + h_L LH_2N + \cdots \tag{1}$$

where $H_{1,2}$ are the usual Higgs doublets, $N$ is the right-handed neutrino component and the ellipsis denotes the supersymmetric standard model part of the superpotential. In order to implement the PQ symmetry, two gauge singlet superfields $\phi_{1,2}$ with PQ charges $q_{1,2}$ are introduced. The structure of the superpotential is determined by the PQ charge assignment: $q_2 = -(n+2)q_1$, $q_{H_1}+q_{H_2} = -(n+1)q_1$ and so on. Obviously the PQ fields $\phi_1$ and $\phi_2$ correspond to flat directions when nonrenormalizable interactions and supersymmetry breaking effects are ignored. This model can be considered as a supersymmetric generalization of the DFSZ axion model (but endowed with a radiative mechanism generating the axion scale) in the sense that the Higgs doublets carry nonzero PQ charges. Note that the second term in the superpotential yields the correct scale for the Higgs mass parameter $\mu = h\langle\phi_1\rangle^{n+1}/M_P^n$ upon spontaneous breaking of the PQ symmetry [10]. Taking into account the radiative effects of the strong Yukawa coupling $h_N NN\phi_1$, the soft mass-squared of $\phi_1$ becomes *negative* at scales around $F_a \simeq \langle\phi_1\rangle$, and thereby driving $\phi_1$ to develop vacuum expectation value at an intermediate scale. This Yukawa coupling is also necessary to keep the field $\phi_1$ in thermal equilibrium at high temperature $T > m_1$ for which $\langle\phi_1\rangle = 0$. Neglecting the field $\phi_2$, the renormalization group improved scalar potential for the singlets is given by

$$V = V_0 - m_1^2|\phi_1|^2 + k^2\frac{|\phi_1|^{2n+4}}{M_P^{2n}}\,, \tag{2}$$

where $m_1^2$ is positive and of order $m_{3/2}^2$, and $V_0$ is a constant of order $m_{3/2}^2F_a^2$ which is introduced to make $V(\langle\phi_1\rangle) = 0$. Clearly the minimum of this scalar potential breaks $U(1)_{PQ}$ by

$$\langle\phi_1\rangle \simeq F_a \simeq (m_{3/2}M_P^n)^{1/n+1}, \tag{3}$$

where we have ignored the coefficients of order unity. The integer $n$ fixes the size of the axion scale. For the smallest value $n = 1$, the axion scale $F_a \simeq \sqrt{m_{3/2}M_P}$ fits into the usual allowed band of the axion scale: $10^{10}\,\mathrm{GeV} \lesssim F_a \lesssim 10^{12}\,\mathrm{GeV}$. Later we will argue that the upper bound on $F_a$ can be relaxed and thus a bigger value of $n$ is allowed also.

The above radiative mechanism generating the axion scale has substantial influence on the history of the universe [6, 11]. At high temperature, $\phi_1$ receives a thermal mass $\delta m_1^2 \simeq |h_N|^2T^2 \gg m_1^2$ leading to $\langle\phi_1\rangle = 0$. This thermal mass is generated by right-handed neutrinos in the thermal bath. Note that the right-handed neutrino $N$ becomes massless when $\langle\phi_1\rangle = 0$ and thus copiously produced when $T \gg m_1$. During this period, $\langle\phi_2\rangle = 0$ also. When the temperature falls below $T \simeq V_0^{1/4}$, which is about $\sqrt{m_{3/2}F_a}$, the universe is dominated by the vacuum energy density $V_0$ and thus there appears a short period of thermal inflation. Below $T < m_1 \simeq m_{3/2}$, the effective mass of $\phi_1$ becomes negative and then $\phi_1$ develops an intermediate scale VEV given by Eq. (3). With $\langle\phi_1\rangle \simeq F_a$, the other flaton field $\phi_2$ develops also a VEV of order $F_a$ through the $A$-type soft SUSY breaking term, $kA\phi_1^{n+2}\phi_2/M_P^n$, in the scalar potential. This procedure makes the thermal inflation end and subsequently the early universe experiences a period dominated by coherently oscillating PQ flaton fields $\phi_1$ and $\phi_2$. More precisely, the oscillating flaton corresponds to a combination of the two complex scalar fields $\phi_1$ and $\phi_2$ which is orthogonal to the axion field $a = \sum_i c_i \arg(\phi_i)$ where $c_i = q_i\langle\phi_i\rangle^2/F_a$.

*NS bound.* After the period of coherent oscillation, the universe would be reheated by the decay products of the oscillating flaton $\varphi$. A feature peculiar to the PQ flatons is that their decay products include axion as one of the main components [7, 11]. The energy density of these axions at the time of nucleosynthesis (NS), $(\rho_a)_{NS}$, should satisfy the conventional nucleosynthesis bound on the extra energy density:

$$\left(\frac{\rho_a}{\rho_\nu}\right)_{NS} \leq \delta N_\nu. \tag{4}$$

Here $\rho_\nu$ denotes the energy density of a single species of relativistic neutrino and $\delta N_\nu$ is the number of extra neutrino species allowed by nucleosynthesis. In the past, $\delta N_\nu$ has been argued to be 0.3 or even smaller as 0.04 [12]. However, although claimed to be quite conservative, more careful recent analyses do not exclude even $\delta N_\nu = 1.5$ [13]. Here we do not take any specific value of $\delta N_\nu$, but examine the implications of the above NS bound for $\delta N_\nu = 0.1 \sim 1.5$

with the hope that one can push $\delta N_\nu$ down to the value 0.1 in the future.

Before evaluating $(\rho_a)_{NS}$, let us first determine the reheat temperature $T_{RH}$ by parameterizing the width of the flaton decay into thermalizable particles as $\Gamma_\varphi = B_a^{-1} M_\varphi^3/64\pi F_a^2$. Here $M_\varphi$ denotes the flaton mass and the prefactor $B_a^{-1}$ will be presumed to be of order 10, which is a proper choice for $(\rho_a)_{NS}$ to satisfy the above NS bound. The reheat temperature is then given by

$$T_{RH} \simeq 1.2 g_{RH}^{-1/4} \sqrt{M_P \Gamma_\varphi} \simeq 1 \left(\frac{0.1}{B_a}\right)^{1/2} \left(\frac{10^{12}\,\mathrm{GeV}}{F_a}\right) \left(\frac{M_\varphi}{300\,\mathrm{GeV}}\right)^{3/2} \mathrm{GeV}\,, \tag{5}$$

where $g_{RH} \equiv g_*(T_{RH})$ counts the effective number of relativistic degree of freedom at $T_{RH}$. The entropy production factor $S_{\rm after}/S_{\rm before}$ for this reheating is of order $V_0/m_{3/2}^3 T_{RH}$ which is roughly of order $10^2 (M_P/m_{3/2})^{(5n-1)/(2n+2)}$. This huge entropy dumping at relatively late time was considered as a promising source for erasing out various unwanted cosmological relics, especially, cosmologically dangerous string moduli [11].

In order to evaluate $(\rho_a)_{NS}$, one needs to know whether axions produced by the late flaton decay have ever been in thermal equilibrium with the thermalized plasma of normal light particles. If axions were in thermal equilibrium at some moment but later frozen out at temperature $T_f$, we would have

$$\left(\frac{\rho_a}{\rho_\nu}\right)_{NS} = \frac{4}{7}\left(\frac{43/4}{g_*(T_f)}\right)^{4/3}. \tag{6}$$

However if axions have never been in equilibrium, $(\rho_a)_{NS}$ is simply determined by the *effective* branching ratio $B_a$ measuring how large fraction of flatons are converted into axions during the reheating. Roughly $B_a \simeq \Gamma_a/\Gamma_{\rm tot}$ with the decay width $\Gamma_a$ of $\varphi \to 2a$, however axions can be produced also by the secondary decays of the decay products of flatons. For unthermalized axions at $T_{RH}$, the ratio between $\rho_a$ and the energy density $\rho_r$ of thermalized radiation would be simply $B_a/(1-B_a)$. We then have

$$\left(\frac{\rho_a}{\rho_\nu}\right)_{NS} = \frac{43}{4}\frac{B_a}{1-B_a}\frac{4}{7}\left(\frac{43/4}{g_{RH}}\right)^{1/3}. \tag{7}$$

In order to see whether axions have ever been in thermal equilibrium, let us consider the axion interaction rate $\Gamma_{\rm int} = \langle\sigma v\rangle N_r$ where $\sigma$ denotes the cross section for the axion scattering off the thermalized radiation with energy density $\rho_r$ and number density $N_r$. A careful look at of the reheating process indicates that $\rho_r \sim R^{-3/2}$, $N_r \sim R^{-1/2}$, and $\rho_\varphi \sim R^{-3} e^{-t\Gamma_{\rm tot}}$ during the

reheating period between $t_0$ and $t_D \simeq \Gamma_{\rm tot}^{-1}$ where $R$ denotes the scale factor and $t_0$ corresponds to the time when the relativistic particles produced by the flaton decay become the major part of the radiation [8]. A simple dimensional analysis implies that the axion cross section can be written as $\sigma = (\gamma_1 + \gamma_2 (m/E)^2)/4\pi F_a^2$, where $E$ denotes the center of mass energy, $\gamma_{1,2}$ are dimensionless constants of order unity or less, and $m$ corresponds to the mass of target particle. We then have $\langle \sigma v \rangle = (\gamma_1 + \gamma_2 (m/\langle E_0 \rangle)^2 (R/R_0)^2)/4\pi F_a^2$. With the informations given above, it is straightforward to see that the ratio $Z = \Gamma_{\rm int}/H$ is an increasing function of $R$ during the reheating period of $t_0 < t < t_D$.

Let us now consider the behavior of $Z = \Gamma_{\rm int}/H$ after the reheating. At $t > t_D$ with $T < T_{RH}$, the entropy production almost ends and thus $N_r \sim R^{-3} \sim T^3$, $H \sim R^{-2} \sim T^2$ as in the standard radiation dominated universe with an adiabatic expansion. Using Eq. (3) with $M_\varphi \simeq m_{3/2}$ and Eq. (5), we find

$$\begin{aligned} \frac{\Gamma_{\rm int}}{H} &= 3 \times 10^{-2} \bar{\gamma} g_*^{1/2} \frac{T M_P}{F_a}^2 \\ &= 10^{-2} \bar{\gamma} \left( \frac{T}{T_{RH}} \right) \left( \frac{g_*}{10^2} \right) \left( \frac{M_\varphi}{M_P} \right)^{3(n-1)/2(n+1)} , \end{aligned} \tag{8}$$

where $\bar{\gamma} = (\gamma_1 + \gamma_2 (m/\langle E \rangle)^2)$.

For $n > 1$, the above result for $t > t_D$ together with the fact that $Z = \Gamma_{\rm int}/H$ is an increasing function of $R$ during $t_0 < t < t_D$ readily implies that $Z \ll 1$ and thus axions have never been in equilibrium. For the case of $n = 1$, we need a bit more discussion about the size of $\bar{\gamma}$. Obviously at tree level, any nontrivial axion couplings to $SU(2) \times U(1)$ *non-singlet* fields arise as a consequence of $SU(2) \times U(1)$ breaking. In other words, tree level axion couplings to normal fields are induced by the mixing with the Higgs doublets. As a result, tree level axion couplings can be described effectively by dimensionless coupling constants which are of order $m/F_a$ where $m$ corresponds to the mass of the particle that couples to the axion. This means that the energy dependent part of the axion cross section, i.e. the $\gamma_2$-part, is due to tree level axion couplings, while the energy independent $\gamma_1$-part is due to the loop-induced axion couplings like $\frac{\alpha_s}{4\pi F_a} a G_{\mu\nu} \tilde{G}^{\mu\nu}$. As a result, $\bar{\gamma}$ is suppressed either by the loop factor $(\frac{1}{8\pi^2})^2$ or by the relativistic factor $(m/\langle E \rangle)^2$. Then we can safely take $\bar{\gamma} \lesssim 1$, implying $Z \ll 1$ for the case of $n = 1$ also.

In the above, we have argued that $Z = \Gamma_{\rm int}/H \ll 1$ and thus axions have never been in thermal equilibrium. Then the axion energy density at nucleosynthesis is given by Eq. (7) and

the NS bound (4) leads to

$$\frac{B_a}{1-B_a} \leq 0.24 \left(\frac{\delta N_\nu}{1.5}\right)\left(\frac{g_{RH}}{43/4}\right)^{1/3}. \tag{9}$$

The above nucleosynthesis limit on $B_a$ depends mildly upon the reheat temperature $T_{RH}$ through the factor $(g_{RH}/g_{NS})^{1/3}$ where $g_{NS} \equiv g_*(T_{NS}) = 43/4$, while it is rather sensitive to the discordant number $\delta N_\nu$ which is presumed here to be in the range $0.1 \sim 1.5$ [12, 13]. For $T_{RH}$ above 0.2 GeV but below the superparticle mass, we have $g_{RH}/g_{NS} = 6 \sim 10$, while $g_{RH}/g_{NS} = 1 \sim 3$ for $T_{RH} < 0.2$ GeV. We thus have just a factor two variation of the limit when $T_{RH}$ varies from the lowest allowed value 6 MeV [16] to the superparticle mass of order 100 GeV. In summary, the NS limit (9) indicates that we need to tune the effective branching ratio $B_a$ to be less than $1/3 \sim 0.02$ for $\delta N_\nu = 0.1 \sim 1.5$.

*Implication on flaton couplings.* We now discuss the implications of the NS bound (9) for generic supersymmetric axion models with a radiative mechanism generating the axion scale. Since the models under consideration involve too many unknown free parameters, we just examine how plausible it is that the NS limit (9) is satisfied for the unknown parameters simply taking their natural values. To proceed, let us write the flaton couplings responsible for the flaton decay as

$$\mathcal{L}_\varphi = \mathcal{L}_{PQ} + \mathcal{L}_{SSM}, \tag{10}$$

where $\mathcal{L}_{PQ}$ describes the couplings to the fields in PQ sector, while $\mathcal{L}_{SSM}$ describes the couplings to the fields in supersymmetric standard model (SSM) sector. Schematically $\mathcal{L}_{PQ}$ is given by

$$\mathcal{L}_{PQ} = \frac{\varphi}{2F_a}\left(M_\varphi^2 a^2 + M_\varphi^2 \varphi'^2 + (M_{\tilde{\varphi}}\tilde{\varphi}\tilde{\varphi} + \text{h.c})\right) \tag{11}$$

where $a$, $\varphi'$, and $\tilde{\varphi}$ denote the axion, other flaton, and flatino respectively. In the above, we have ignored the model-dependent dimensionless coefficients of each terms which are of order unity in general.

The flaton couplings to SSM fields are more model dependent. In DFSZ type models, flaton couplings to the SSM sector are essentially due to the mixing with the Higgs doublets. Then flaton couplings can be read off by making the replacement

$$H_i \to v_i + \frac{x_{ij}v_j}{F_a}\varphi, \tag{12}$$

where $v_i = \langle H_i \rangle$, $x_{ij}$'s are model-dependent coefficients which are generically of order unity. Then again schematically

$$\mathcal{L}_{SSM} = \frac{\varphi}{F_a} \left\{ (M_1 \chi\chi' + M_2 \chi\lambda + \text{h.c.}) + M_3^2 A_\mu A^\mu + M_4^2 zz' + M_5^2 |z|^2 \right\}, \tag{13}$$

where $z$ and $z'$ denote spin zero fields in the SSM, e.g. squarks, sleptons and Higgs, with their fermionic partners $\chi$ and $\chi'$, while $(A^\mu, \lambda)$ stands for the gauge multiplets which become massive due to the Higgs doublets VEVs, i.e $W$ and $Z$. The order of magnitude estimate of the dimensionful coefficients leads to: $M_1 \simeq M_\chi$, $M_2 \simeq M_3 \simeq M_W$, $M_4^2 \simeq M_\chi(A + \mu \cot\beta)$, and $M_5^2 \simeq M_\chi^2 + M_W^2 \cos 2\beta$, where $M_\chi$ and $M_W$ denote the masses of $\chi$ and $W$ respectively, $\tan\beta = v_2/v_1$, and again we have ignored the coefficients of order unity.

As is well known, besides DFSZ type models, there are another interesting class of axion models named as hadronic axion models. In hadronic axion models, all SSM fields carry *vanishing* PQ charge and as a result flaton couplings to SSM fields appear as loop effects. As an example of supersymmetric hadronic axion model with a mechanism generating the axion scale radiatively, let us consider a model with

$$W = k \frac{\phi_1^{n+2} \phi_2}{M_P^n} + h_Q Q Q^c \phi_1 + \cdots, \tag{14}$$

where $\phi_{1,2}$ are gauge singlet flatons, and $Q$ and $Q^c$ stand for additional heavy quark and anti-quark superfields. Again the soft mass-squared of $\phi_1$ becomes *negative* at scales around $F_a$ by the radiative corrections involving the strong Yukawa coupling $h_Q Q Q^c \phi_1$, thereby generating the axion scale as $\langle \phi_1 \rangle \simeq \langle \phi_2 \rangle \simeq F_a$. A peculiar feature of this type of hadronic axion models is that at tree level flatons do not couple to SSM fields, while there are nonzero couplings to PQ fields as Eq. (11). Flaton couplings to SSM fields are then induced by the loops of $Q$ and $Q^c$, yielding

$$\mathcal{L}_{SSM} = \frac{\alpha_s}{2\pi} \frac{\varphi}{F_a} \left( \frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu} + i \bar{\lambda}^a \gamma \cdot \partial \lambda^a ) \right), \tag{15}$$

where $(G^a_{\mu\nu}, \lambda^a)$ denotes the gluon supermultiplet (and possibly other gauge multiplets) and again we ignored dimensionless coefficients of order unity.

It is now easy to notice that, due to the loop suppression in $\mathcal{L}_{SSM}$, most of oscillating flatons in hadronic axion models decay first into either axion pairs, or lighter flaton pairs, or flatino pairs, as long as the decays are kinematically allowed. Lighter flatons would experience similar decay modes, while flatinos decay into axion plus a lighter flatino. Then in the first

round of reheating, flatons are converted into either axions or the lightest flatinos. The lightest flatinos will eventually decay into SSM particles. Because of kinematical reasons, e.g. the mass relation $M_\varphi > 2M_{\tilde{\varphi}}$ and the phase space suppression factor $(1 - 4M^2_{\tilde{\varphi}}/M^2_\varphi)^{1/2}$ in the decay $\varphi \to 2\tilde{\varphi}$, more than half of the original flatons would be converted into axions, i.e. the effective branching ratio $B_a \gtrsim 1/2$, *unless* the flaton coupling to the *lightest* flatino is unusually large. This is in conflict with the NS limit (9) even for the most conservative choice $\delta N_\nu = 1.5$, implying that hadronic axion models with a radiative mechanism can be compatible with the big-bang nucleosynthesis only when the models are tuned to have an unusually large flaton coupling to the lightest flatino.

In DFSZ type models, flatons have tree level couplings to SSM fields which are of order $M_{\rm SSM}/F_a$ or $M^2_{\rm SSM}/F_a$ where $M_{\rm SSM}$ collectively denotes the mass parameters in the SSM, e.g. $M_t$, $M_W$, $\mu$, $A$, and so on [see Eq. (13) and the discussions below it]. Thus if $M_\varphi \gg M_{\rm SSM}$, the reheating procedure would be similar as that of hadronic axion models and then the NS limits provides a meaningful constraint on the flaton couplings to the PQ sector. For the case that $M_\varphi$ is comparable to $M_{\rm SSM}$, the most conservative choice of $\delta N_\nu = 1.5$ would not provide any meaningful restriction on DFSZ type models. However it is still nontrivial to achieve $B_a$ significantly smaller than 1/10. A careful examination of the flaton couplings in DFSZ type models suggests that, among the decays into SSM particles, the decay channels to the top ($t$) and/or stop ($\tilde{t}$) pairs are most important. Flaton coupling to the top (stop) is of order $M_t/F_a$ ($M^2_{\tilde{t}}/F_a$), while the coupling to the axion is of order $M^2_\varphi/F_a$. As a result, $B_a$ significantly smaller than 1/10 implies that the flaton couplings to the top and/or stop are unusually large in view of the relation $M_\varphi > 2M_t$ ($2M_{\tilde{t}}$). One of the efficient way to achieve such a small $B_a$ is to assume that there is a sort of mass hierarchy between the lighter stop mass-squared $M^2_{\tilde{t}_1}$ and the heavier stop-mass squared $M^2_{\tilde{t}_2}$, allowing for instance $4M^2_{\tilde{t}_1} < M^2_\varphi < \frac{1}{4}M^2_{\tilde{t}_2}$. This would be the case when $M^2_{\tilde{t}} + M^2_t \simeq M^2_{\tilde{t}^c} + M^2_t \simeq M_t(A + \mu\cot\beta)$ where $M^2_{\tilde{t}}$ and $M^2_{\tilde{t}^c}$ denote the soft squark masses. Since the flaton couplings to stops are determined not only by the mass parameters (e.g. $M_t$ and $A$) but also by additional dimensionless parameters $x_{ij}$ defined in Eq. (12), the flaton coupling to the lighter stop $\tilde{t}_1$ would be of order $M^2_{\tilde{t}_2}/F_a$, not the order of $M^2_{\tilde{t}_1}/F_a$. To be more explicit, let us write this coupling as $x_1 M^2_{\tilde{t}_2}\varphi|\tilde{t}_1|^2/F_a$. With the

flaton-axion coupling given by $x_a M_\varphi^2 \varphi a^2/2F_a$, we find

$$\frac{\Gamma_a}{\Gamma_{\tilde{t}_1}} = \frac{1}{32}\left(\frac{x_a}{x_1}\right)^2 \left(\frac{2M_\varphi}{M_{\tilde{t}_2}}\right)^4 \left(1-\frac{4M_{\tilde{t}_1}^2}{M_\varphi^2}\right)^{-1/2} . \tag{16}$$

This shows that $B_a$ can be smaller than about $10^{-2}$ for the parameter range: $x_a \approx x_1$ and $4M_{\tilde{t}_1}^2 < M_\varphi^2 < \frac{1}{4}M_{\tilde{t}_2}^2$.

*Relaxation of the bound on $F_a$.* The reheat temperature can not be arbitrarily low in order to be compatible with the big bang nucleosynthesis. Since flatons produce large number of hardrons, the bound $T_{RH} > 6$ MeV has to be B met [14]. With Eq. (5), this leads to the upper bound:

$$F_a \lesssim 2\times 10^{14}\left(\frac{0.1}{B_a}\right)^{1/2}\left(\frac{M_\varphi}{300\,\mathrm{GeV}}\right)^{3/2}\ \mathrm{GeV}\,. \tag{17}$$

Once one uses the relation $F_a \simeq (M_\varphi M_P^n)^{1/n+1}$, this means that only $n = 1$, 2, and 3 are allowed by the big-bang nucleosynthesis.

As is well known, another upper bound on the axion scale can be derived by requiring that the coherent axion energy density produced by an initial misalignment should not exceed the critical density [15]. If there is no entropy production after the axion start to oscillate at around $T \simeq 1$ GeV, this lead to the usual bound: $F_a \lesssim 10^{12}$ GeV. When $n = 2$ or 3, the corresponding axion scale $F_a \simeq (M_\varphi M_P^n)^{1/n+1}$ would exceed this bound. However in this case, the reheat temperature (5) goes below 1 GeV. Then the coherent axions may be significantly diluted by the entropy dumped from flaton decays, thereby allowing $F_a$ much bigger than $10^{12}$ GeV [8].

Axion production in matter-dominated universe, e.g. flaton oscillation dominated universe, has been considered in Ref. [14, 16] assuming $m_a(T) \propto T^{-4}$. For our computation, we take the power-law fit of the temperature dependent axion mass [17]:

$$m_a(T) \simeq 7.7\times 10^{-2}\, m_a(T\!=\!0)(\Lambda_{QCD}/T)^{3.7}\,.$$

Axion oscillation starts at $T_a$ for which $m_a(T_a) = 3H(T_a)$:

$$T_a \simeq 0.9\left(\frac{\Lambda_{QCD}}{200\,\mathrm{MeV}}\right)^{0.48}\left(\frac{M_\varphi}{300\,\mathrm{GeV}}\right)^{0.39}\left(\frac{10^{12}\,\mathrm{GeV}}{F_a}\right)^{0.39}\ \mathrm{GeV}\,. \tag{18}$$

We refer the reader to paper [18] for the available formulae. If $T_a > T_{RH}$, the coherent axion energy density is diluted by the entropy produced between $T_a$ and $T_{RH}$. At the end of the

entropy dumping (around $T_{RH}$), the coherent axion number density in unit of the entropy density is given by $Y_f \simeq \theta^2 F_a^2 m_a(T_a) R_a^3/S_f$ where $\theta$ denotes the initial misalignment angle of the axion field, $R_a$ is the scale factor at $T_a$ and $S_f$ is the total entropy at $T_{RH}$. The ratio of the axion energy density to the critical energy density at present is given by

$$\begin{aligned}\Omega_a h_{50}^2 &\simeq 3.3\times 10^{17}\left(\frac{F_a}{10^{12}\,\mathrm{GeV}}\right)^{1.5}\left(\frac{\Gamma_\varphi}{\mathrm{GeV}}\right)^{0.98}\left(\frac{\Lambda_{QCD}}{200\ \mathrm{MeV}}\right)^{-1.9}\\ &\simeq 1\left(\frac{0.1}{B_a}\right)\left(\frac{10^{12}\,\mathrm{GeV}}{F_a}\right)^{0.44}\left(\frac{M_\varphi}{300\,\mathrm{GeV}}\right)^{2.9}\left(\frac{\Lambda_{QCD}}{200\ \mathrm{MeV}}\right)^{-1.9}\end{aligned} \qquad (19)$$

where we have used $\Gamma_\varphi \simeq B_a^{-1} M_\varphi^3/32\pi F_a^2$. The above result is valid only for $n \geq 2$ yielding $T_{RH} < T_a$. As we have anticipated, it shows that the case of $n=2$ or 3 with $F_a \simeq (M_\varphi M_P^n)^{1/n+1}$ yields a coherent axion energy density not exceeding the critical density although the corresponding $F_a$ exceeds $10^{12}$ GeV. Furthermore, in this case of $n=2$ or 3, *axions can be a good dark matter candidate* for an appropriate value of $M_\varphi$, which was not possible for $n=1$.

We remark that diluting the coherent axions with $T_{RH} < T_a$ is allowed only when R-parity is broken. If not, stable lightest supersymmetric particles (LSP) produced after the flaton decay would overclose the universe. This can be avoided if the reheat temperature is bigger than the decoupling temperature of LSP which is typically $M_{LSP}/20$. However this is usually above 1 GeV, i.e. above $T_a$. Consequently, the usual upper bound $F_a \lesssim 10^{12}$ GeV can not be relaxed when the reheat temperature is bigger than $M_{LSP}/20$. We stress here that even when R-parity is broken and thus LSP cannot be a dark matter candidate, coherent axions can be a viable dark matter candidate when $n=2$ or 3.

*Baryogenesis.* Thermal inflation driven by PQ flatons may dilute away any pre-existing baryon asymmetry. However, PQ flatons themselves can produce baryon asymmetry after the reheating through the DH mechanism [9]. A complicated Affleck-Dine type baryogenesis after thermal inflation has also been explored in Ref. [19]. Our previous discussion of flaton couplings in DFSZ type models indicates that flatons going to stops can be the most efficient decay channel. The decay-produced stops subsequently decay to generate a baryon asymmetry provided that the baryon-number violating operator, e.g., $\lambda''_{332} U_3^c D_3^c D_2^c$ and the corresponding complex trilinear soft-term are present. Note that the PQ symmetry [see Eq. (1)] can be arranged so that dangerous lepton-number violating operators $LQD^c, LLE^c$ are forbidden for the proton stability.

In order for the baryon asymmetry not to be erased the reheat temperature (5) has again to be less than few GeV [9]. This again means that the DH mechanism can work only for $n = 2$ or 3 [see Eqs. (3) and (5)]. The produced baryon asymmetry is

$$\eta \equiv \frac{n_B}{n_\gamma} \simeq 5.3 \frac{T_{RH}}{M_\varphi} \Delta B \,, \tag{20}$$

where $\Delta B$ is the baryon asymmetry generated by each flaton decay into stop-antistop pair. Using Eq. (5) and the estimate of $\Delta B$ given in [9], we find

$$\frac{\eta}{3 \times 10^{-10}} \simeq |\lambda''_{332}|^2 \left( \frac{\arg(A m^*_{1/2})}{10^{-2}} \right) \left( \frac{0.1}{B_a} \right)^{1/2} \left( \frac{10^{14}\,\mathrm{GeV}}{F_a} \right) \left( \frac{M_\varphi}{300\,\mathrm{GeV}} \right)^{1/2} , \tag{21}$$

where $\arg(A m^*_{1/2})$ denotes the CP violating relative phase which is constrained to be less than $10^{-2}$ for superparticle masses of order 100 GeV [20]. For $n = 3$, the desired amount of baryon asymmetry can be achieved only when $\lambda''_{332}$ is of order unity, while for $n = 2$ it can be done with a smaller $\lambda''_{332}$.

In conclusion, we have examined some cosmological consequences of supersymmetric axion models in which the axion scale is radiatively generated as $F_a \simeq (m_{3/2} M_P^n)^{1/n+1}$. In such models, the early universe inevitably experiences a period dominated by the coherent oscillation of PQ flatons which start to oscillate at temperature around $m_{3/2}$. Then a significant amount of oscillating PQ flatons can decay into axions, thereby yielding a too large axion energy density at the time nucleosynthesis. This consideration puts a limit on the effective branching ratio $B_a$ measuring how large fraction of oscillating flatons are converted into axions: it should be less than $1/3 \sim 0.02$ depending upon our choice of the allowed extra number of neutrino species $\delta N_\nu = 0.1 \sim 1.5$. Models of hadronic axion with a radiative mechanism would yield $B_a \gtrsim 0.5$ unless the flaton coupling to the lightest flatino is unusually large. This is essentially because the flaton couplings to SSM are loop suppressed compared to the couplings to PQ sector. DFSZ type models with a radiative mechanism is more interesting since it can provide a rationale for the size of the $\mu$ term (and also the scale for neutrino masses). If the flaton mass $M_\varphi \gg M_{\rm SSM}$ denoting the typical mass in supersymmetric standard model, DFSZ type models would also suffer from the same difficulty as that of hadronic axion models. However for $M_\varphi$ comparable to $M_{\rm SSM}$, requiring $B_a$ to be about $1/10$ does not provide any meaningful constraint on DFSZ type models. If one wishes to achieve a smaller $B_a$, say about $10^{-2}$ in DFSZ type models, one then needs a kind of tuning of the model. Flaton decays into the

lighter stops is then picked out as one of the efficient decay channels leading to such a small $B_a$ provided $4M_{\tilde{t}_1}^2 < M_\varphi^2 < \frac{1}{4}M_{\tilde{t}_2}^2$.

Another interesting cosmological consequence of decaying flatons is the relaxation of the cosmological upper bound on the axion scale. For the axion scale bigger than $10^{12}$ GeV, the entropy production by PQ flatons ends after the axion field starts to oscillate by QCD instanton effects, thereby diluting the coherent axion energy density in a rather natural way. With this late time entropy production by PQ flatons, the upper bound on the axion scale $F_a$ can be pushed up to about $10^{15}$ GeV, but at the expense of breaking R-parity to avoid a too large mass density of relic LSP. Then the integer $n$ which determines the axion scale in terms of $m_{3/2}$ and $M_P$ can take $n = 1$, 2 or 3.

It is likely that any pre-existing baryon asymmetry is completely diluted by the huge entropy dumping in thermal inflation scenario. As the PQ flatons are expected to decay dominantly into stops, the DH mechanism for the late time baryogenesis can work in a natural manner when $n = 2$ or 3 so that the reheat temperature does not exceed 1 GeV. With broken R-parity, LSP is no more stable and can not be a dark matter candidate. In this scenario, coherent axions can provide a critical mass density of the universe by saturating the cosmological bound on $F_a$ which now can be as large as $10^{15}$ GeV.

Interestingly enough, we now observe that the case of $n = 2$ or $n = 3$ provides a very concordant cosmological scenario: (i) a proper baryon asymmetry is generated by the DH mechanism using baryon-number violating interaction $\lambda'' U^c D^c D^c$, (ii) potentially dangerous coherent axions (with $F_a \gg 10^{12}$ GeV) are diluted by the late time entropy production, (iii) both the baryogenesis and axion dilution require R-parity to be broken, and then diluted coherent axions constitute dark matter in the universe,

**Acknowledgments**: This work is supported in part by KOSEF through CTP of Seoul National University (KC, JEK), KAIST Basic Science Research Program (KC), KOSEF Grant 951-0207-002-2 (KC), Basic Science Research Institutes Program, BSRI-96-2418 (JEK), BSRI-96-2434 (KC), and the SNU-Nagoya Collaboration Program of Korea Research Foundation (KC, EJC, JEK). EJC is a Brain-Pool fellow.

# References

[1] R. D. Peccei and H. R. Quinn, Phys. Rev. Lett. 38 (1977) 1440 ; Phys. Rev. D 16 (1977) 1791 .

[2] J. E. Kim, Phys. Rev. Lett. 43 (1979) 103; M. A. Shifman, V. I. Vainstein and V. I. Zakharov, Nucl. Phys. B 166 (1980) 493.

[3] A. R. Zhitnitskii, Sov. J. Nucl. Phys. 31 (1980) 260; M. Dine, W. Fischler and M. Srednicki, Phys. Lett. B 104 (1981) 199.

[4] For a review see, J. E. Kim, Phys. Rep. 150 (1987) 1; E. W. Kolb and M. S. Turner, *The Early Universe* (Addison-Wesley, 1990).

[5] H. Murayama, H. Suzuki and T. Yanagida, Phys. Lett. B 291 (1992) 418.

[6] D. H. Lyth and E. D. Stewart, Phys. Rev. Lett. 75 (1995) 201, and references therein.

[7] E. J. Chun and A. Lukas, Phys. Lett. B 357 (1995) 43.

[8] M. Dine and W. Fischler, Phys. Lett. B 120 (1983) 137; P. J. Steinhardt and M. S. Turner, Phys. Lett. B 129 (1983) 51.

[9] S. Dimopoulos and L. J. Hall, Phys. Lett. B 196 (1987) 135.

[10] J. E. Kim and H. P. Nilles, Phys. Lett. B 138 (1984) 150; E. J. Chun, J. E. Kim and H. P. Nilles, Nucl. Phys. B 370 (1992) 105.

[11] D. H. Lyth and E. D. Stewart, Phys. Rev. D 53 (1996) 1784.

[12] T. P. Walker, G. Steigman, D. N. Schramm, K. A. Olive and H. Kang, Astrophys. J. 376 (1991) 51; P. Kernan and L. Krauss, Phys. Rev. Lett. 72 (1994) 3309; K. A. Olive and G. Steigman, Phys. Lett. B 354 (1995) 357.

[13] C. J. Copi, D. N. Schramm and M. S. Turner, Phys. Rev. Lett. 75 (1995) 3981, and FERMILAB-Pub-96/122-A, astro-ph/9606059; P. J. Kernan and S. Sarkar, CWRU-P3-96, astro-ph/9603045; C. Y. Cardall and G. M. Fuller, astro-ph/9603071.

[14] G. Lazarides, R. K. Schaefer, D. Seckel and Q. Shafi, Nucl. Phys. B346 (1990) 193.

[15] J. Preskill, M. Wise and F. Wilczek, Phys. Lett. B 120 (1983) 127; L. Abbott and P. Sikivie, *ibid* (1983) 133; M. Dine and W. Fischler, *ibid* (1983) 137.

[16] G. Lazarides, C Panagiotakopoulos and Q. Shafi, Phys. Lett. B 192 (1987) 323.

[17] M. S. Turner, Phys. Rev. D 33 (1986) 889; D. J. Gross, R. D. Pisarski and L. G. Yaffe, Rev. Mod. Phys. 53 (1981) 43.

[18] R. J. Scherrer and M. S. Turner, Phys. Rev. D 31 (1985) 681.

[19] E. D. Stewart, M. Kawasaki and T. Yanagida, RESCEU-8-96, hep-ph/9603324.

[20] J. Ellis, S. Ferrara and D. V. Nonopoulos, Phys. Lett. B 114 (1982) 231; W. Buchmueller and D. Wyler, Phys. Lett. B 121 (1983) 321; J. Polchinski and M. Wise, Phys. Lett. B 125 (1983) 393; F. del Aguila, M. Gavela, J. Grifols and A. Mendez, Phys. Lett. B 126 (1983) 71.